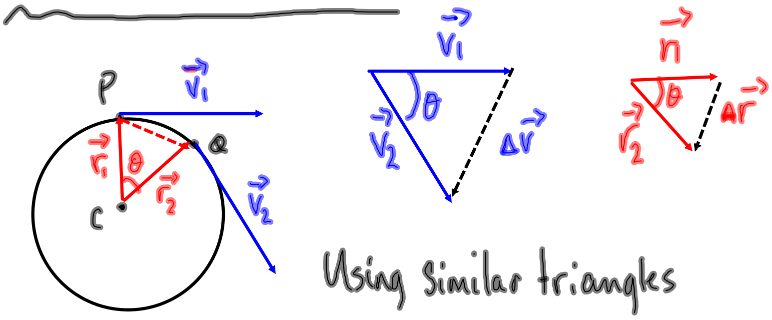
§11.2 Uniform Circular Motion



Using similar triangles

$|\Delta\vec{r}| = \Delta r \quad |\Delta\vec{v}| = \Delta v$

$|\vec{r}_1| = |\vec{r}_2| = r \quad |\vec{v}_1| = |\vec{v}_2| = v$

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

The distance actually travelled:

$\Delta d = v\Delta t$

But as $\Delta t \to 0$, $\Delta r \to \Delta d$

$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

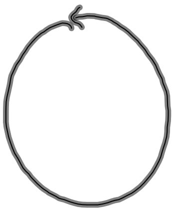
$$\frac{v^2}{r} = \frac{\Delta v}{\Delta t}$$

centre-seeking

Centripetal acceleration (magnitude only)

$$a_c = \frac{v^2}{r}$$

Dir ⇒ always to the centre of the curved path.



$\Delta d = 2\pi r$

$\Delta t = T$ (period)

$v = \frac{\Delta d}{\Delta t}$

$$v = \frac{2\pi r}{T}$$ ← tangential velocity

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{4\pi^2 r^2}{T^2 r}$$

$T = \frac{1}{f}$

$$a_c = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

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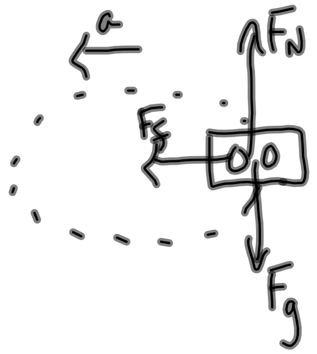
$m = 2125\,kg$

$r = 52m$

$\mu = 0.70$

$V_{max} = ?$



$F_{net} = ma$

$F_f = m\frac{v^2}{r}$

$\mu F_N = \frac{mv^2}{r}$

$\mu mg = \frac{mv^2}{r}$

$\mu g = \frac{v^2}{r}$

$v^2 = \mu g r$

$v^2 = (0.70)(9.81\,m/s^2)(52m)$

$\boxed{V \doteq 19\,m/s}$

If you travel faster than 19m/s, you will not be able to maintain the curved path.

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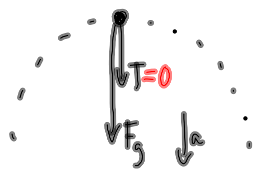
$m = 225\,g$

$r = 1.2\,m$

a) $V_{min} = ?$ ($T = 0$) ← tension

b) $T = ?$ (bottom)

a) At the top:



$$\vec{F}_{net} = m\vec{a}$$

$$F_g = m\frac{v^2}{r}$$

$$mg = m\frac{v^2}{r}$$

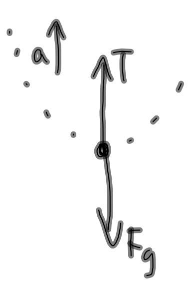
$$v^2 = gr$$

$$v^2 = (9.81\,m/s^2)(1.2\,m)$$

$$V = 3.4\,m/s$$

If you don't swing fast enough, the yo-yo will not complete the loop.

b) At the bottom (maximum tension)



$$\vec{F}_{net} = m\vec{a}$$

$$T - F_g = m\frac{v^2}{r}$$

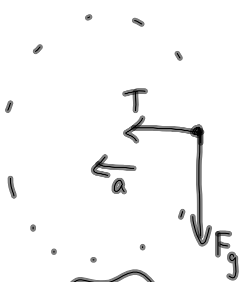
$$T = \frac{mv^2}{r} + mg$$

$$T = \frac{(0.225\,kg)(3.4\,m/s)^2}{1.2\,m} + (0.225\,kg)(9.81\,m/s^2)$$

$$T = 2.2\,N + 2.2\,N$$

$$T = 4.4\,N$$

at the side:



$$\vec{F}_{net} = m\vec{a}$$

$$T = \frac{mv^2}{r}$$

$$T = 2.2\,N$$

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